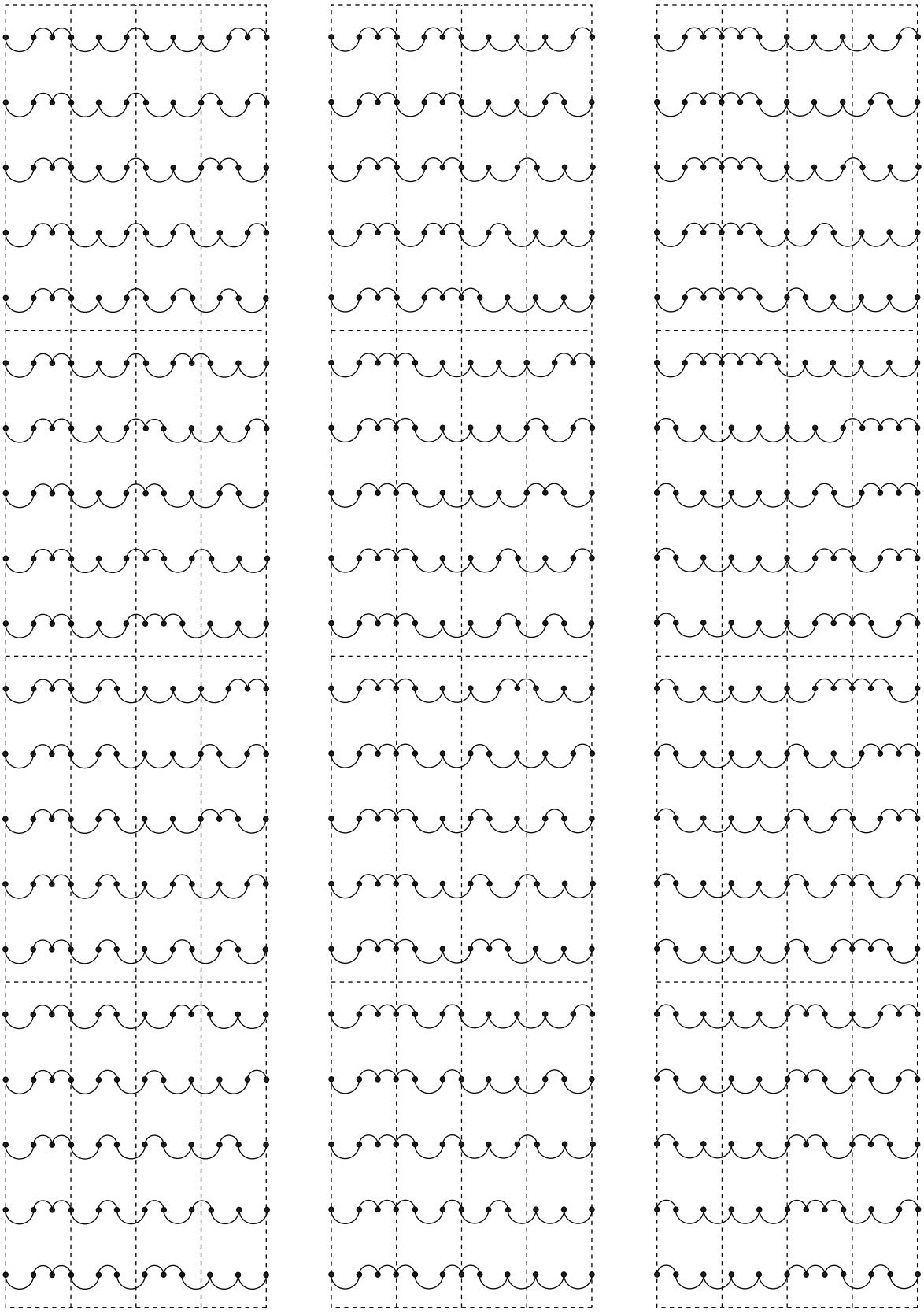
Basic Rhythm (a combinatorial approach)



Miles Okazaki

FILLING FOUR BEATS

In music all over the world, the most common cycle is four beats.
A cycle is something that repeats, and beats are pulses that we feel in time.
A repeating four beat cycle can be counted "one, two, three, four, one, two, three, four...", etc.
Most people who listen to music will have heard four beat cycles thousands of times.
This study takes a look at some ways to explore four beats, and the interesting mathematical patterns that result from a few simple rules.

Imagine these boxes as four beats:

1	2	3	4

Say: "one, two, three, four"

This is not particularly interesting, but we can do more things with these four beats if we break them into smaller pieces. Let's break each one in half:

1	&	2	&	3	&	4	&

Say: "one and two and three and four and"

It's the same space, but divided into more parts. Eight parts, in this case. You can see right away that if we divided each beat into three parts, we would get 12 parts, four parts leads to 16, and so on.

Here is the game: We are going to fill the four beat space without gaps or overlaps, using some very simple building blocks, to see what kind of patterns emerge. Patterns need different types of elements in order to be interesting. For example, a chessboard would not look like much if all the squares are the same color. So, like a chessboard, we will use two building blocks to have some variation, but in order to be rhythmically interesting, they need to be different lengths (instead of colors).
We will use rhythmic building blocks from the first two prime numbers: 2 and 3.

Question 1: How many ways can you fill eight parts with the numbers 2 and 3?

Answer: Four (2,2,2,2) (3,3,2) (3,2,3) (2,3,3)

Now this can easily be done with numbers, but we want to translate it into music, to get a feeling for how these numbers "sound." In order to do this, we are going to borrow two rhythmic translations of the numbers 2 and 3 from the Carnatic music of South India:

2 = "taka"
3 = "takita"

The four rhythms we came up with before now become:

ta	ka	ta	ka	ta	ka	ta	ka

ta	ki	ta	ta	ki	ta	ta	ka

ta	ki	ta	ta	ka	ta	ki	ta

ta	ka	ta	ki	ta	ta	ki	ta

These are four ways to fill in four beats with syllables of lengths 2 and 3.

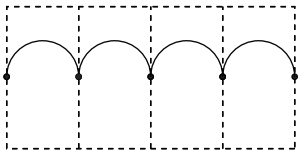
Clap four beats and say these syllables.
The beginning of each building block is colored red, to see more easily.
Try accenting (saying louder) these red syllables.

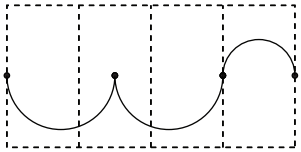
You may notice that these written out syllables are a little bit hard to read.
But the syllables themselves are very easy to memorize (there are only two of them).
So, let's memorize "taka" and "takita," and use symbols for them instead:

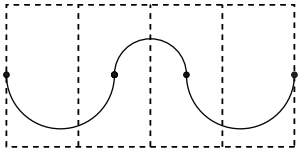
taka = 

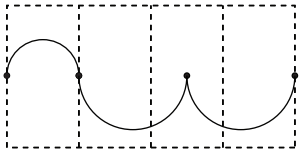
takita = 

Imagine that "taka" is smaller, light, standing up and "takita" is larger, heavy, hanging down.
Now our four syllable patterns become visual patterns:

taka taka taka taka 

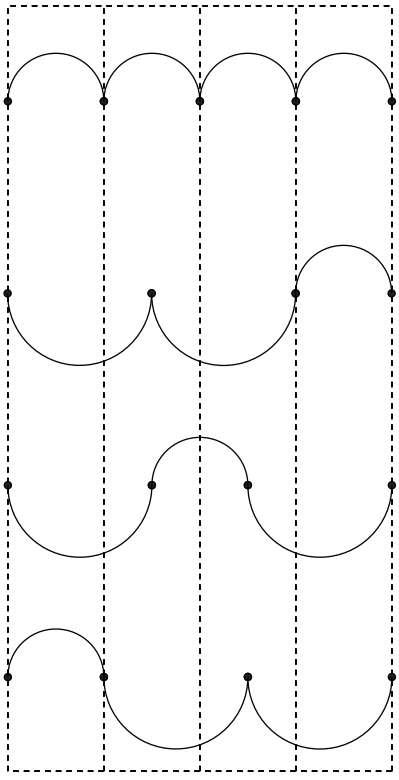
takita takita taka 

takita taka takita 

taka takita takita 

Let's put them all together into one table:

2 (1 - 4)



This table is saying:

1) Each beat is divided into two parts (the number 2 in the box)
2) There are four patterns (the "1 - 4" listing after the number 2)

This is a symbolic way of writing "taka" and "takita," intuitive and simple enough that you could do it with a pencil and paper, or just mentally.

Question: So, what is the next step?

Answer: Dividing the beats into smaller parts.

Question: How many patterns can you make when each beat is divided into three parts?

Answer: (next page)

Dividing the beat into three parts gives the rhythm a different feeling.
As a shortcut, we'll call this triple division "Tisra." Four beats divided into three parts each makes 12 parts. By trial and error, you can find that there are twelve ways to fill four *tisra* beats with *taka* and *takita*. Mathematically, this is the same as saying that there are twelve ways to make the numbers 2 and 3 add up to the number 12:

2 + 2 + 2 + 2 + 2 + 2 = 12
3 + 3 + 2 + 2 + 2 = 12
3 + 2 + 3 + 2 + 2 = 12
3 + 2 + 2 + 3 + 2 = 12
3 + 2 + 2 + 3 + 2 = 12
2 + 3 + 3 + 2 + 2 = 12
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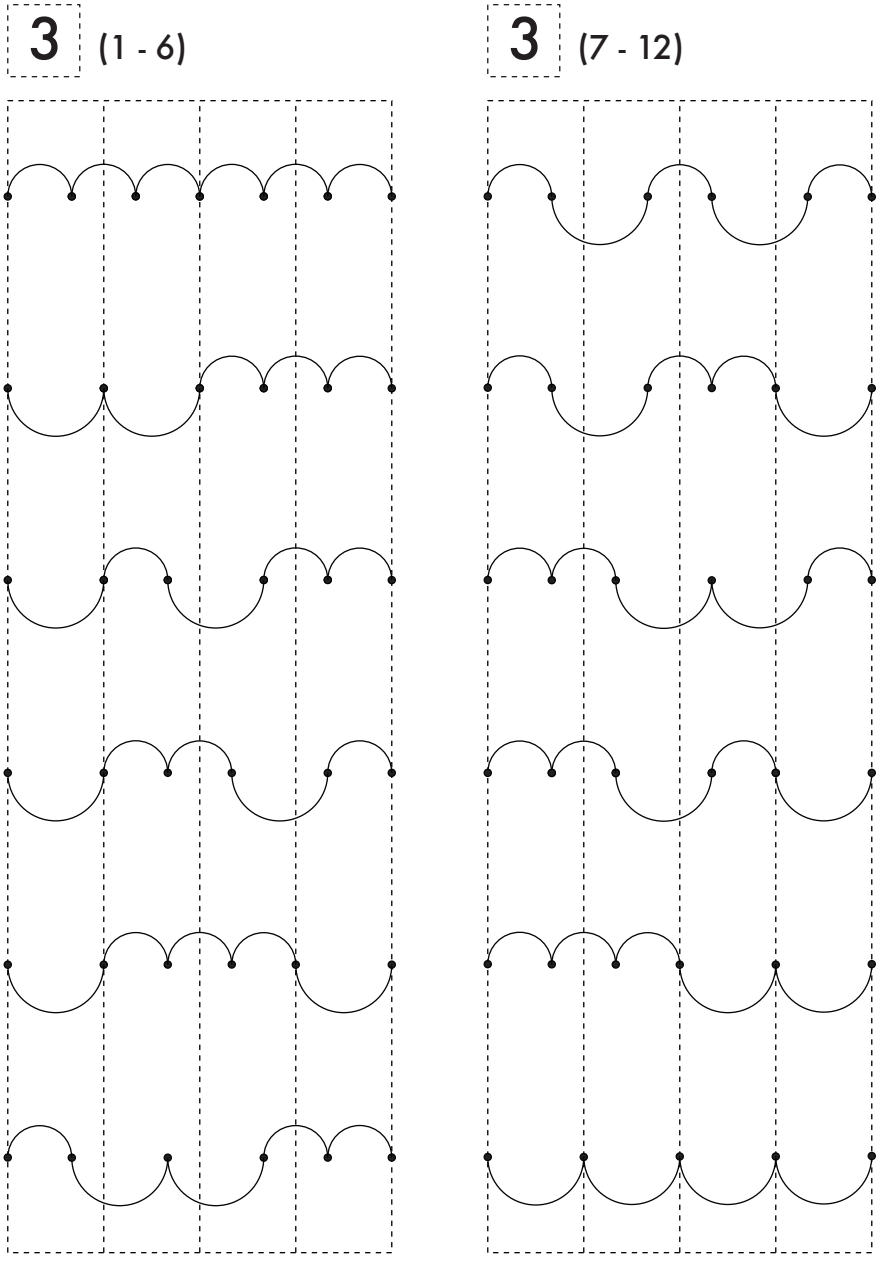
Here is the same thing visualized:



You can see that all of these patterns are made of one of three possible combinations of *taka* and *takita*:

1) All *taka* (first pattern)
2) All *takita* (last pattern)
3) Two *takita* and three *taka* (all other patterns)

In this last case, any order that we put the syllables in will result in the same length. This is the same as the commutative property of addition.

Even though these different orderings of *taka* and *takita* take the same amount of space, they sound different from each other when we perform them.

Try saying the syllables for each of these while clapping the four beats and feeling three parts within each beat.

Notice how the sound changes as you move through the combinations.

Question: How many combinations do you get with division of the beat into four parts?

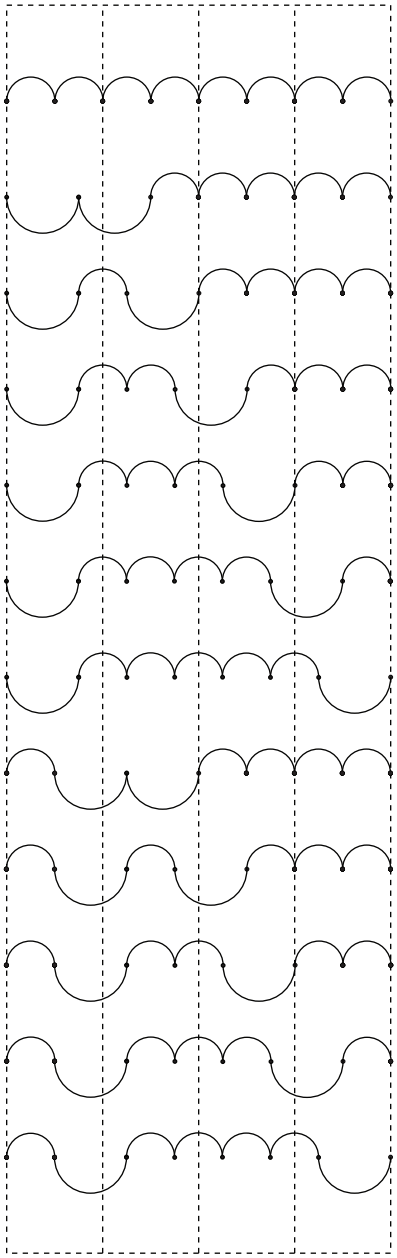
Answer: (next page)

Answer: 37

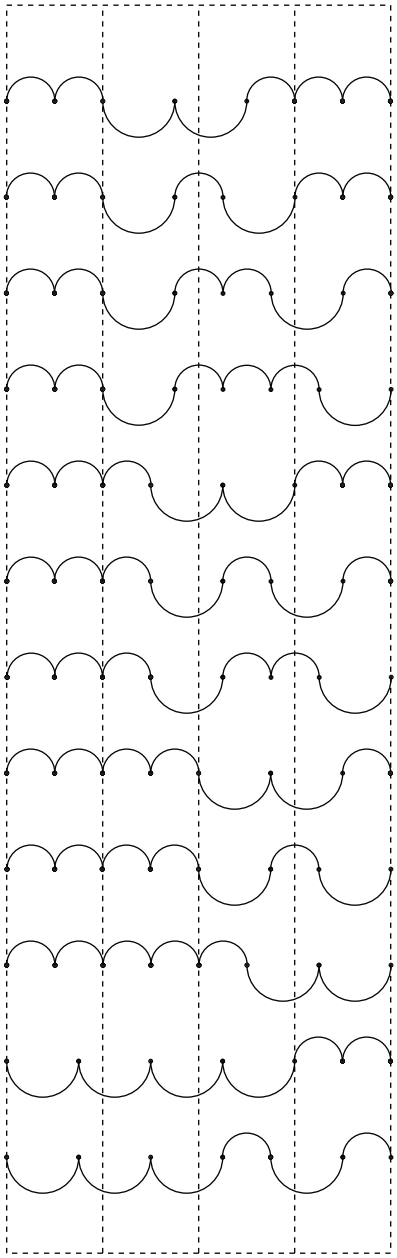
Dividing the beat into 4 parts gives us a total of 16 units to group into twos and threes.
We'll call this type of feel *chatusra*.
In *chatusra*, four beats can be filled with all *taka* (first pattern), two *takita* and five *taka* (patterns 2-22), or four *takita* and two *taka* (patterns 23-37).

Try saying the syllables for these patterns while clapping four beats and feeling four parts per beat.

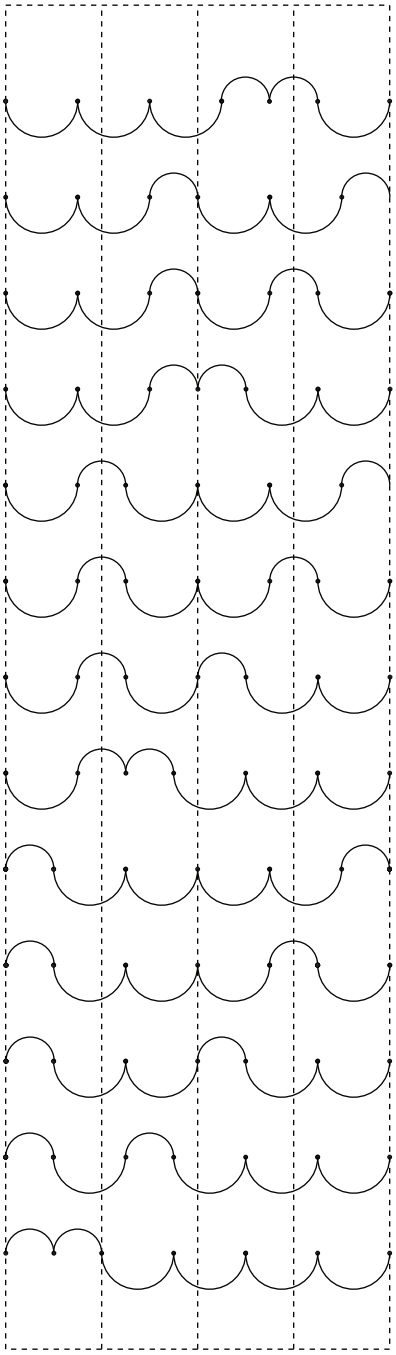
4 (1 - 12)



4 (13 - 24)



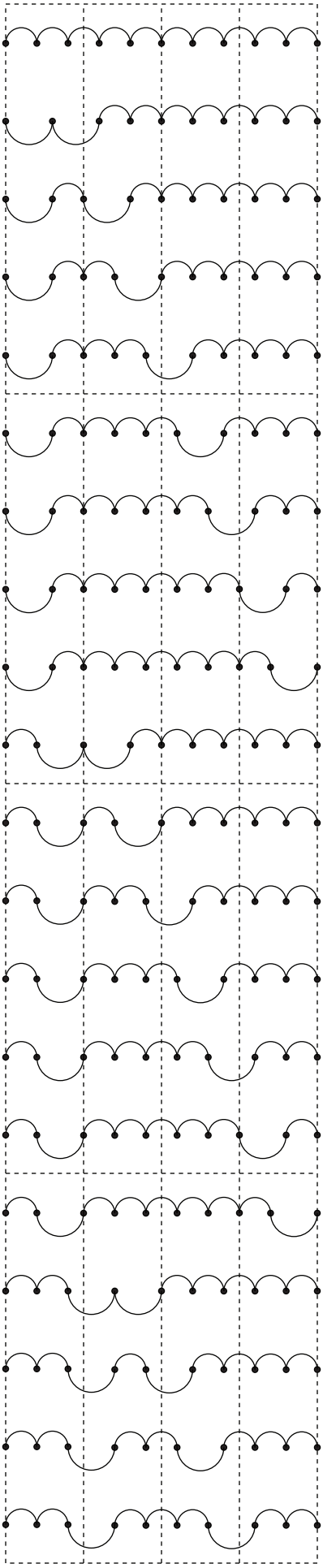
4 (25 - 37)



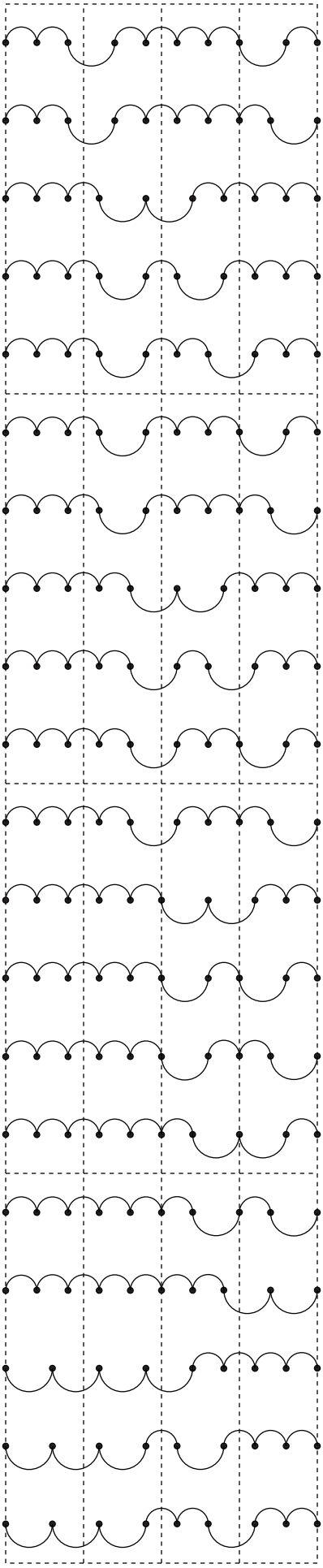
Question: How many patterns are possible with division of the beat into five parts (*khanda*)?

Answer: Next two pages

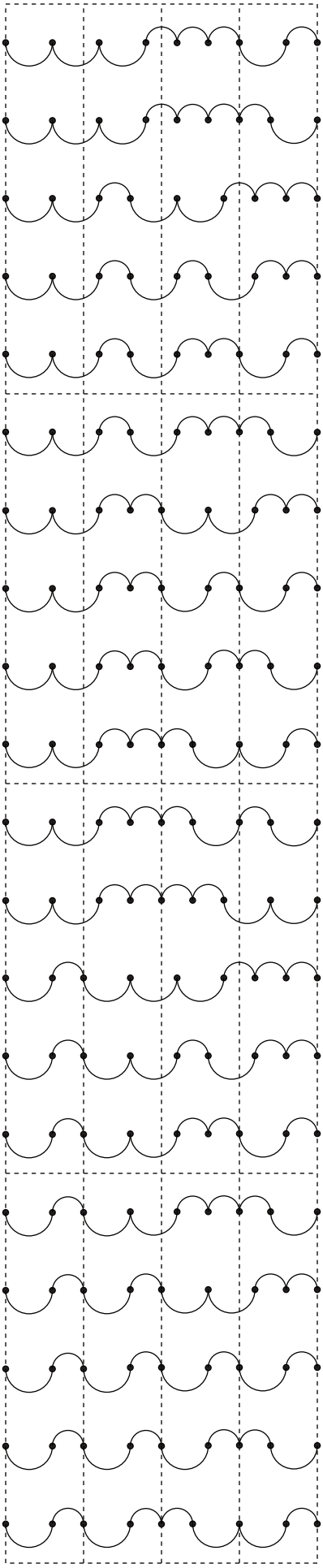
5 (1 - 20)



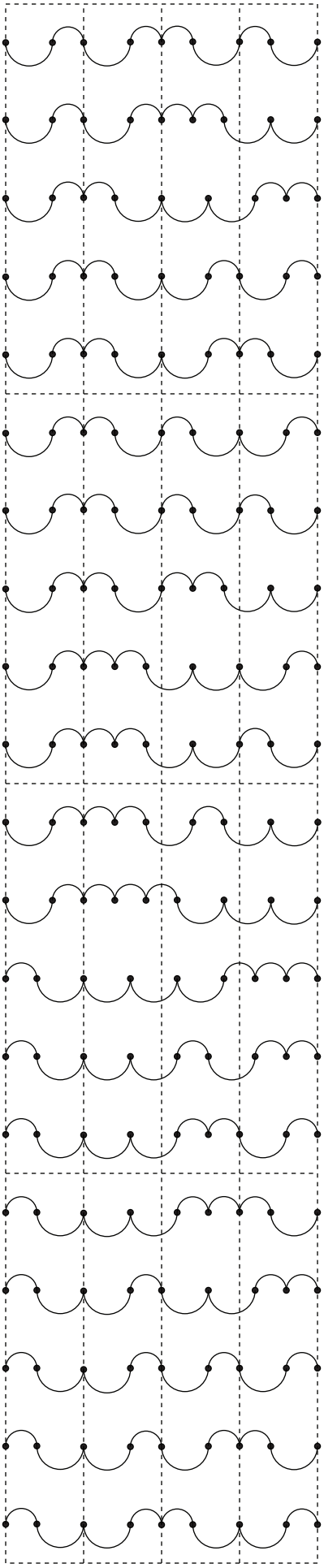
5 (21 - 40)



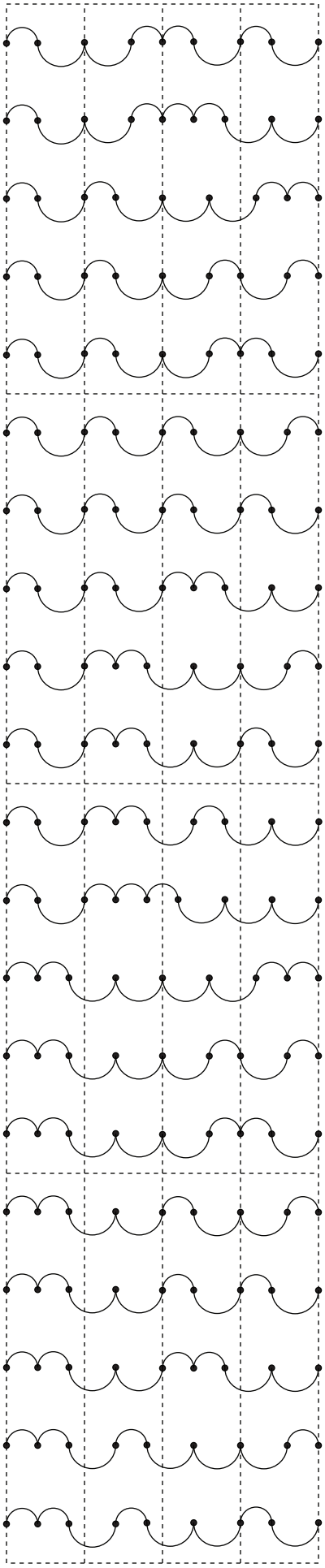
5 (41 - 60)



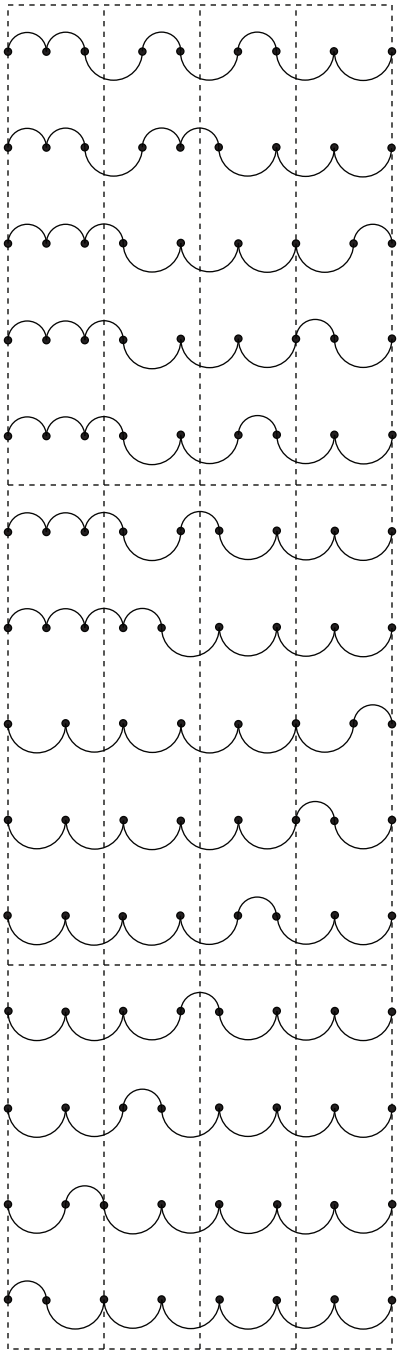
5 (61 - 80)



5 (81 - 100)



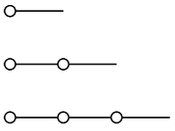
5 (101 - 114)



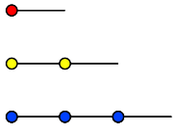
These exercises are going to be more difficult than the previous ones. But they are still just four beats long. Each time you divide the beat into smaller parts, you have to say the syllables faster to fit them into the four beats. Try divisions of 6, 7 and 8 parts. The number of possible combinations will be 351, 1081, and 3329.

RHYTHMIC CONSTRUCTION KIT

Imagine the numbers 1, 2, and 3 as dots and lines:



Add colors to see them more clearly:

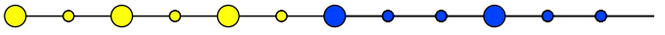


These are our three building blocks. Let's make some constructions.
Here is (1,2,3):

Here is (2,1,3,1):

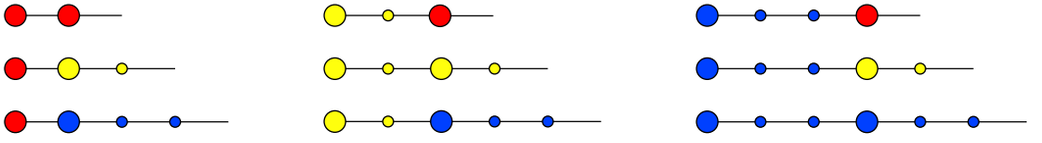
Here is (2,2,2,3,3):

This last one doesn't really work. It's hard to tell the difference between the twos and threes.
To fix this, make the first dot of each group larger:



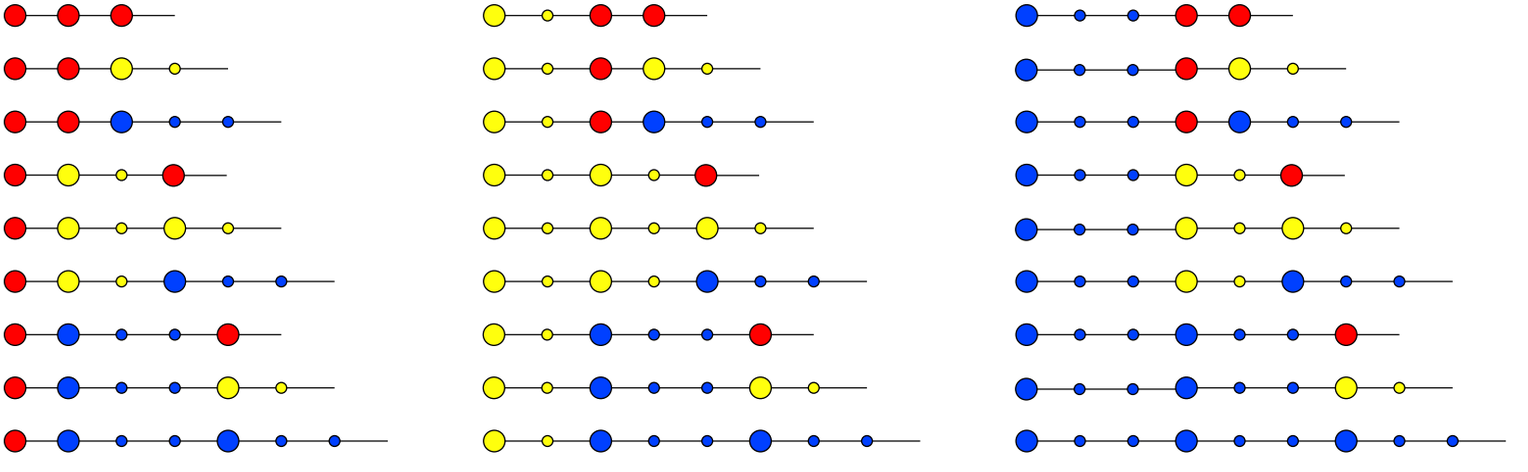
Question 1: Using only two numbers, how many different constructions can you make?

Answer: 9



Question 2: Using three numbers, how many different constructions can you make?

Answer: 27



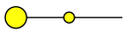
Question 3:

Based on the patterns above, how many constructions would we be able to make with 4, 5, and 6 numbers?

What is the general rule that you are using to find this number?

Now we can make musical rhythms with these constructions.
Imagine that these building blocks have sounds of 1, 2 and 3 syllables:

Ta 

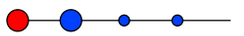
Taka 

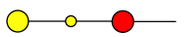
Takita 

Here are the syllable versions of our first 9 constructions:

TaTa 

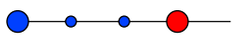
TaTaka 

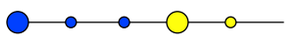
TaTakita 

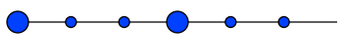
TakaTa 

TakaTaka 

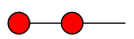
TakaTakita 

TakitaTa 

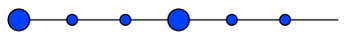
TakitaTaka 

TakitaTakita 

Notice that some of these are repetitive. Meaning, they say the same thing twice:

TaTa 

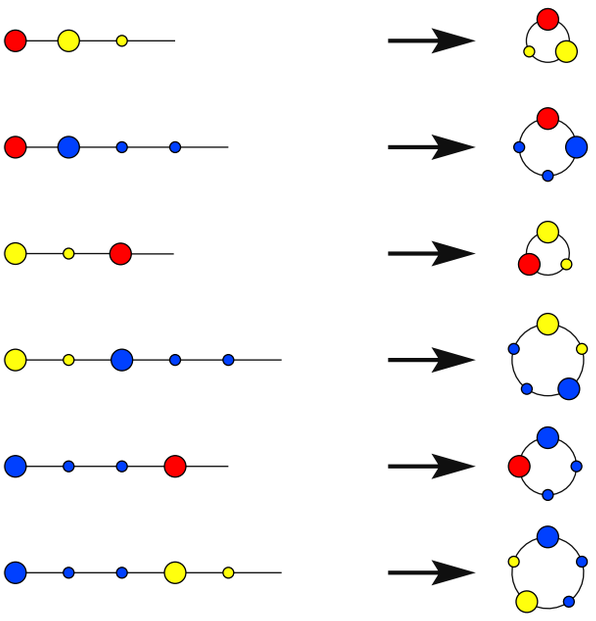
TakaTaka 

TakitaTakita 

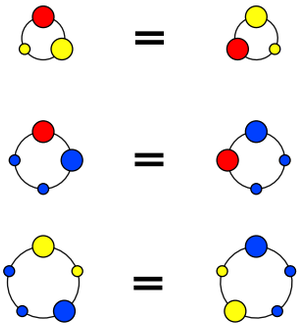
If we're trying to build unique rhythms (that is, things we haven't seen before), we can take these off the list. They "sound" the same as the original building blocks, just repeated.
This leaves us with 6 unique rhythmic constructions made out of two numbers.

Exercise 1: Write and say the syllable versions of the 27 constructions on the previous page, removing any repetitive rhythms.

Now for the next step. In music, rhythms are often played in cycles, which means that they repeat over and over.
Common terms for a repeating rhythmic shape include “groove,” “pattern,” or “loop.”
To visualize this, imagine bending our first six unique rhythmic constructions into circular loops.



Notice that, when seen as a circle, some of them are the same as others in rotation:

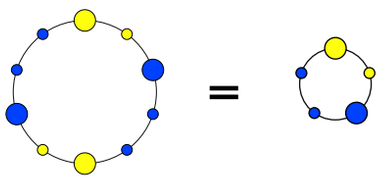


This means that, if we use the circular version, we can take three more rhythms off the list. This leaves us with only three unique rhythms made of two parts:

Question 1: How many unique circular rhythms can you make with three parts?

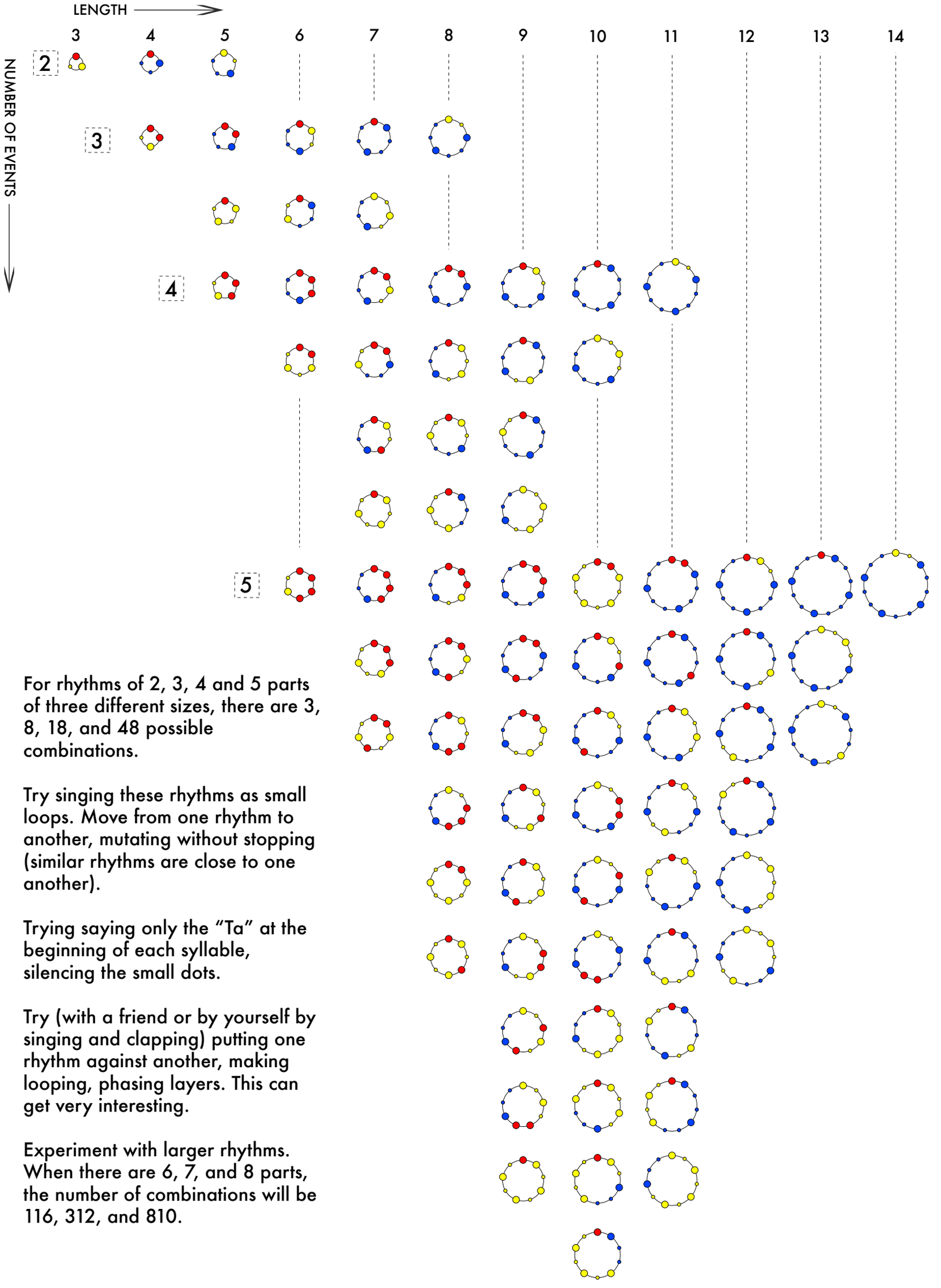
Question 2: How many unique circular rhythms can you make with four parts?

Note: be careful on this one. You will run into the problem of repetition in a new way.
For example, the rhythm TakaTakitaTakaTakita “sounds” the same as “TakaTakita” repeated twice.
This could be visualized like this:



Question 3: How many unique circular rhythms can you make with five parts?

(answers on the next page)



For rhythms of 2, 3, 4 and 5 parts of three different sizes, there are 3, 8, 18, and 48 possible combinations.

Try singing these rhythms as small loops. Move from one rhythm to another, mutating without stopping (similar rhythms are close to one another).

Trying saying only the “Ta” at the beginning of each syllable, silencing the small dots.

Try (with a friend or by yourself by singing and clapping) putting one rhythm against another, making looping, phasing layers. This can get very interesting.

Experiment with larger rhythms. When there are 6, 7, and 8 parts, the number of combinations will be 116, 312, and 810.